

# ENVI-met 3.0: Updated Model Overview

March 2004

Michael Bruse

## 1. INTRODUCTION

This document describes the microscale model ENVI-met Version 3.0 (Bruse, 1999; Bruse and Fleer, 1998). It replaces older versions of the same name and should be used in combination with other papers available on the website [www.envi-met.com](http://www.envi-met.com) describing several aspects more in detail. In particular, it is an update of the article published in *Environmental Modelling and Software* and should be used instead.

## 2 THE ATMOSPHERIC MODEL

This section describes the main prognostic variables in the atmospheric model. These variables are the main wind flow, temperature, humidity and turbulence:

### 2.1 Mean Air Flow

The three-dimensional turbulent air flow in the model is given by the non-hydrostatic incompressible Navier-Stokes equations (1-a) - (1-c):

$$\frac{\partial u}{\partial t} + u_i \frac{\partial u}{\partial x_i} = - \frac{\partial p}{\partial x} + K_m \left( \frac{\partial^2 u}{\partial x_i^2} \right) + f(v - v_g) - S_u \quad (1 \text{ a})$$

$$\frac{\partial v}{\partial t} + u_i \frac{\partial v}{\partial x_i} = - \frac{\partial p}{\partial y} + K_m \left( \frac{\partial^2 v}{\partial x_i^2} \right) - f(u - u_g) - S_v \quad (1 \text{ b})$$

$$\frac{\partial w}{\partial t} + u_i \frac{\partial w}{\partial x_i} = - \frac{\partial p}{\partial z} + K_m \left( \frac{\partial^2 w}{\partial x_i^2} \right) + g \frac{\theta(z)}{\theta_{ref}(z)} - S_w \quad (1 \text{ c})$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

where  $f (=10^4 \text{ sec}^{-1})$  is the Coriolis parameter,  $p$  is the local pressure perturbation and  $\theta$  the potential temperature at level  $z$ . The reference temperature  $\theta_{ref}$  represents the larger scale meteorological conditions and is calculated as an average temperature over all grid cells of height  $z$ , excluding those occupied by buildings.

The air density  $\rho$  was removed from the original compressible Navier-Stokes equations using the *Boussinesq-Approximation*, which leads to one additional source term in the w-equation to include thermal forced vertical motion and one continuity (filter) equation (2) which has to be satisfied for each time step in order to keep the flow field mass conserving. (Note that all three-dimensional advection and diffusion terms are written in Einstein summation ( $u_i=u,v,w$ ;  $x_i=x,y,z$  for  $i=1,2,3$  to save place)

The local source/sink terms  $S_u$ ,  $S_v$  and  $S_w$  describe the loss of wind speed due to drag forces occurring at vegetation elements. Following Liu (1996) and Yamada (1982) this effect can be parameterized as

$$S_{u(i)} = \frac{\partial \bar{p}'}{\partial x_i} = c_{d,f} LAD(z) \cdot W \cdot u_i \quad (3)$$

where  $W = (u^2 + v^2 + w^2)^{0.5}$  is the mean wind speed at height  $z$ ,  $LAD(z)$  is the leaf area density in [ $m^2 m^{-3}$ ] of the plant in this height. The mechanical drag coefficient at plant elements  $c_{d,f}$  is set to 0.2.

**Boundary conditions:** A *no-slip* condition is used for all solid surfaces. The inflow profile is obtained from the one-dimensional reference model and a zero-gradient Neumann condition is used at the outflow and lateral boundaries. At the top boundary all vertical motions are assumed to be zero. Special boundary conditions are used for the pressure perturbation on all outflow boundaries to keep the model mass conserving.

## 2.2 Temperature and Humidity

The distribution of the air temperature  $\theta$  and specific humidity  $q$  is given by the combined advection-diffusion equation with internal source/sinks :

$$\frac{\partial \theta}{\partial t} + u_i \frac{\partial \theta}{\partial x_i} = K_h \left( \frac{\partial^2 \theta}{\partial x_i^2} \right) + \frac{1}{c_p \rho} \frac{\partial R_{n,lw}}{\partial z} + Q_h \quad (4)$$

$$\frac{\partial q}{\partial t} + u_i \frac{\partial q}{\partial x_i} = K_q \left( \frac{\partial^2 q}{\partial x_i^2} \right) + Q_q \quad (5)$$

Similar to the momentum equations,  $Q_h$  and  $Q_q$  are used to link heat and vapour exchange at plants with the atmospheric model. The quantity of  $Q_h$  and  $Q_q$  is provided by the vegetation model described later on.  $\partial R_{n,lw} / \partial z$  is the vertical divergence of longwave radiation taking into account the cooling and heating effect of radiative fluxes.

**Boundary conditions:** The surface temperature of the ground surfaces, of roofs and of walls are used as real physical boundaries. For the inflow profile, Dirichlet, Neuman or cyclic boundary conditions can be selected. At the outflow and lateral boundaries a zero-gradient condition is used. The values for the top of the three dimensional model are obtained from the one dimensional boundary layer model, which extends up to 2500 m.

## 2.3 Atmospheric turbulence

Turbulence is produced when the air flow is sheared at building walls or vegetation elements. Under windy conditions, the magnitude of local turbulence production normally surpasses its dissipation, so that turbulent eddies are transported by the mean air flow. Depending on the structure of the flow, this leads to an increased turbulence away from the original source of disturbance.

To simulate this effect, a so-called 1.5 order turbulence closure model is used in ENVI-met. Based on the work of Mellor and Yamada (1975) two additional prognostic variables, the local turbulence ( $E$ ) and its dissipation rate ( $\epsilon$ ) are added to the model. Their distribution is given by the prognostic equation set:

$$\begin{aligned}\frac{\partial E}{\partial t} + u_i \frac{\partial E}{\partial x_i} &= K_E \left( \frac{\partial^2 E}{\partial x_i^2} \right) + Pr - Th + Q_E - \varepsilon \\ \frac{\partial \varepsilon}{\partial t} + u_i \frac{\partial \varepsilon}{\partial x_i} &= K_\varepsilon \left( \frac{\partial^2 \varepsilon}{\partial x_i^2} \right) + c_1 \frac{\varepsilon}{E} Pr - c_3 \frac{\varepsilon}{E} Th - c_2 \frac{\varepsilon^2}{E} + Q_\varepsilon\end{aligned}\quad (6,7)$$

The terms  $Pr$  and  $Th$  describe the production and dissipation of turbulent energy due to wind shear and thermal stratification,  $Q_E$  and  $Q_\varepsilon$  are the local source terms for turbulence production and dissipation at vegetation.

The mechanical production  $Pr$  is parameterized using the three-dimensional deformation tensor of the local wind field:

$$Pr = K_m \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \quad \text{with } i,j=1,2,3 \quad (8)$$

The buoyancy production  $Th$  is given by

$$Th = \frac{g}{\theta_{ref}(z)} K_h \frac{\partial \theta}{\partial z} \quad (9)$$

To calibrate the  $\varepsilon$ -equation, standard values  $c_1=1.44$ ,  $c_2=1.92$  and  $c_3=1.44$  given by Launder and Spalding (1974) have been used. It has to be noted, that the application of the 1.5 order closure model to the atmospheric boundary layer has certain uncertainties. Depending on the specific situation, different calibration values might be used and the production of turbulent energy normally needs to be restricted in the higher layers of the atmosphere.

Following Liu *et al.* (1996) and Wilson (1988), two extra source terms are added to the  $E$ - $\varepsilon$  System to consider the additional turbulence produced at vegetation as well as the turbulence destruction due to the cascade from larger shear-induced eddies to smaller and waker eddies:

$$Q_E = c_{d,f} LAD(z) \cdot W^3 - 4c_{d,f} LAD(z) \cdot |W| \cdot E \quad (10)$$

$$Q_\varepsilon = 1.5c_{d,f} LAD(z) \cdot W^3 - 6c_{d,f} LAD(z) \cdot |W| \cdot \varepsilon \quad (11)$$

where  $W$  is the mean wind speed like in (3). The source term for the dissipation equation (11) is based on the Kolmogorov relation (Launder and Spalding, 1974) and should be adjusted by measured data if available (see e.g. Liu *et al.*, 1996).

From the calculated  $E$ - $\varepsilon$  field the turbulent exchange coefficients are calculated assuming local turbulence isotropy using the relationships

$$K_m = c_\mu \frac{E^2}{\varepsilon}; K_H, K_q = 1.35 \cdot K_m; K_E = \frac{K_m}{\sigma_E}; K_\varepsilon = \frac{K_m}{\sigma_\varepsilon} \quad (12 \text{ a-d})$$

with  $c_\mu=0.09$ ,  $\sigma_E=1$  and  $\sigma_\varepsilon=1.3$ .

**Boundary conditions:** At all solid surfaces  $E$  and  $\varepsilon$  are calculated as a function of local tangential friction velocity  $u^*$  calculated using the flow components tangential to the concerned surface:

$$E(z=0), E_w = \frac{(u_*^2)^{\tan}}{\sqrt{c_\mu}}, \quad \varepsilon(z=0), \varepsilon_w = \frac{(u_*^3)^{\tan}}{\kappa \cdot z_0}$$

with  $k$ : von-Kármán constant ( $=0.4$ ) and  $z_0$ : microscale roughness length of the surface.

## 2.4 Radiative Fluxes

As a boundary condition, the incoming shortwave and longwave fluxes are needed at the model top. Those are provided using a two-stream radiative flux approximation for the longwave fluxes and a set of empirical equations for the shortwave wavelength spectra (Taesler and Anderson 1984; Gross 1991).

Inside the three-dimensional model, the radiative fluxes are modified by plants and buildings. To estimate their effect on the radiative conditions, the concept of flux reduction coefficients ( $\sigma_{...}$ ) ranging from 1 for undisturbed fluxes to 0 for a total absorption is used (Bruse 1995).

In total, five different reduction coefficients are defined:

$$\begin{aligned}
 \text{(I)} \quad & \sigma_{\text{sw,dir}}(z) = \exp(-F \cdot \text{LAI}^*(z)) \\
 \text{(II)} \quad & \sigma_{\text{sw,dif}}(z) = \exp(-F \cdot \text{LAI}(z, z_p)) \\
 \text{(III)} \quad & \sigma_{\text{lw}}^\downarrow(z) = \exp(-F \cdot \text{LAI}(z, z_p)) \\
 \text{(IV)} \quad & \sigma_{\text{lw}}^\uparrow(z) = \exp(-F \cdot \text{LAI}(0, z)) \\
 \text{(V)} \quad & \sigma_{\text{svf}}(z) = 1/360 \sum_{\pi=0}^{360} \cos \lambda(\pi)
 \end{aligned} \tag{13 a-e}$$

These coefficients describe the influence of vegetation on direct and diffuse shortwave radiation (I and II) and on the downward and upward flux of longwave radiation (III and IV). Coefficient (V) parameterises the local obstruction of the sky by buildings („*Sky-View-Factor*“) and ranges from 1 (free sky) to 0 (no sky visible) where  $\lambda$  is the maximum shielding angle found by the ray-tracing module in direction  $\pi$ .

LAI is the one-dimensional vertical leaf area index of the plant from level  $z$  to the top of the plant at  $z_p$  or the ground  $z=0$ :

$$\text{LAI}(z, z + \Delta z) = \int_{z'}^{z' + \Delta z} \text{LAD}(z') dz'$$

To calculate the decrease of the direct solar radiation, the three-dimensional index LAI\* is used instead of the one-dimensional vertical LAI. LAI\* is calculated with respect to the angle of incidence from the incoming sun rays and analyses the model environment for objects intersecting with the ray path. If a building is found to lie between the point of interest and the sun,  $\sigma_{\text{sw,dir}}$  is set to zero immediately (=shaded), if vegetation is found, the intensity is adjusted as shown in (13 a).

The direct and diffuse shortwave radiation fluxes at any point can then be calculated as

$$\begin{aligned}
 R_{\text{sw,dir}}(z) &= \sigma_{\text{sw,dir}}(z) R_{\text{sw,dir}}^0 \\
 R_{\text{sw,dif}}(z) &= \sigma_{\text{sw,dif}}(z) \sigma_{\text{svf}}(z) R_{\text{sw,dif}}^0 + (1 - \sigma_{\text{svf}}(z)) R_{\text{sw,dir}}^0 \cdot \bar{a}
 \end{aligned} \tag{14 a,b}$$

where  $R_{\text{sw,dir}}^0$  and  $R_{\text{sw,dif}}^0$  are the direct and diffuse shortwave radiative fluxes at the model top. The additional last term for the diffuse component considers the reflection of shortwave radiation inside the environment using the average wall albedo ( $\bar{a}$ ) as reflectivity indicator.

In case of the longwave radiation (14 c-e) it is assumed that shielding vegetation layers will absorb parts of the flux and replace it with their own longwave radiation. Horizontal longwave radiation fluxes from building walls (14 e) are calculated by weighting the emitted radiation of the walls with the sky-view-factor. Using the concept of reduction coefficients, the longwave fluxes at level  $z$  are:

$$\begin{aligned} R_{lw}^{\downarrow}(z) &= \sigma_{lw}^{\downarrow}(z)R_{lw}^{\downarrow,0} + (1 - \sigma_{lw}^{\downarrow}(z))\epsilon_f \sigma_B \bar{T}_{f+}^4 \\ R_{lw}^{\uparrow}(z) &= \sigma_{lw}^{\uparrow}(z)\epsilon_s \sigma_B T_0^4 + (1 - \sigma_{lw}^{\uparrow}(z))\epsilon_f \sigma_B \bar{T}_{f-}^4 \\ R_{lw}^{\leftrightarrow}(z) &= (1 - \sigma_{svf}(z))\epsilon_w \sigma_B \bar{T}_w^4 \end{aligned} \quad (14 \text{ c,d,e})$$

with:

$\bar{T}_{f+}^4, \bar{T}_{f-}^4$ : average foliage temperature of the overlying (+) and underlying (-) vegetation layer,

$T_0$ : ground surface temperature

$\bar{T}_w$ : average surface temperature building walls

$\epsilon_f, \epsilon_s, \epsilon_w$ : emissivity of foliage, the ground surface and of the walls

$\sigma_B$ : Stefan-Boltzman constant.

### 3. THE SOIL MODEL

It is typical for urban environments that a wide range of different soil and surface type can be found varying from natural soils to completely artificial materials. To simulate these heterogenous situations, individual soil properties such as thermodynamic and hydraulic conductivity or albedo, can be assigned to each grid cell of the surface/ soil model.

The soil model is organised in 14 layers between the surface and its lower boundary in 2 m depth. The vertical resolution varies between 0.01 m close to the surface and 0.5 m in the deeper layers. The exchange processes are simulated in terms of heat and water transfer between the layers. Except of the uppermost soil layer in which the heat transfer is calculated in three dimensions, the soil is treated as a one dimensional vertical column. The distribution of heat  $T$  and soil volumetric moisture content  $\eta$  are given by the one dimensional prognostic equations:

$$\frac{\partial T}{\partial t} = \kappa_s \frac{\partial^2 T}{\partial z^2} \quad (15)$$

$$\frac{\partial \eta}{\partial t} = D_\eta \frac{\partial^2 \eta}{\partial z^2} + \frac{\partial K_\eta}{\partial z} - S_\eta(z) \quad (16)$$

For natural soils, the thermal diffusivity  $\kappa_s$  is a function of the available soil moisture  $\eta$  and is calculated after Tjernström (1989). The hydraulic parameters used in (16) are: volumetric water content  $\eta$ , its saturation value  $\eta_s$ , the hydraulic conductivity  $K_\eta$  and the hydraulic diffusivity  $D_\eta$ . All coefficients are calculated using the equations given by Clapp and Hornberger (1978).

As an additional factor, the water uptake by the plant roots ( $S_\eta$ ) provided by the vegetation model has to be considered as an internal sink of moisture. Further more, the evaporation of the soil surface as given by (26 c) has to be considered as an external sink (or source in case of condensation) at the top layers of the soil model.

### 4. THE VEGETATION MODEL

Vegetation is treated as a one-dimensional column with height  $z_p$  in which the profile of leaf area density (LAD) is used to describe the amount and the distribution of leafs. The same concept is used inside the soil system: the distribution of roots is represented by the root area density (RAD) profile stretching from the surface down to the root depth  $-z_r$ . This scheme is universal and can be used for small plants like grass or crop as well as for huge trees if  $z_p$  and  $-z_r$  are adjusted accordingly.

#### 4.1 Turbulent fluxes of heat and vapour

The interactions between the plant leafs and the surrounding air can be expressed in terms of sensible heat flux ( $J_{f,h}$ ), evaporation flux of liquid water on the leafs ( $J_{f,evap}$ ) and transpiration flux controlled by the leaf stomata ( $J_{f,trans}$ ):

$$\begin{aligned} J_{f,h} &= l r_a^{-1} (T_f - T_a) \\ J_{f,evap} &= r_a^{-1} \Delta q \delta_c f_w + r_a^{-1} (1 - \delta_c) \Delta q \\ J_{f,trans} &= \delta_c (r_a + r_s)^{-1} (1 - f_w) \Delta q \end{aligned} \quad (17a,b,c)$$

$T_a$  and  $q_a$  are the temperature and the specific humidity of the air around the leaf,  $\Delta q$  is the leaf-to-air humidity deficit with  $\Delta q = q^*(T_f) - q_a$ .  $T_f$  is the foliage temperature and  $q^*$  the saturation value of  $q$  at the leaf surface. Following Barden (1982), the aerodynamic resistance  $r_a$  is a function of the leaf geometry and wind speed:

$$r_a = A \sqrt{\frac{D}{\max(W, 0.05)}} \quad (18)$$

where  $W$  = wind speed at the leaf surface. The parameter  $A$  is  $87 \text{ sec}^{0.5} \text{m}^{-1}$  for conifers and grass and  $200 \text{ sec}^{0.5} \text{m}^{-1}$  for deciduous trees.  $D$  is the typical leaf diameter ranging from 0.02 m for conifers up to 0.5 m or more for tropical plants (Schilling, 1990). The max condition ensures that no invalid values appear in the case of very low winds.

The factor  $\delta_c$  is set to 1 if evaporation and transpiration can occur ( $\Delta q \geq 0$ ), otherwise  $\delta_c$  is 0 and only condensation is possible. Assuming that only wet parts of the vegetation can evaporate (17b) and, on the other side, only dry parts will transpire (17c), the fraction of wet leaves inside one grid box is needed. Following Deardorff (1978) the wet fraction can be calculated as

$$f_w = \left( \frac{W_{dew}}{W_{dew,max}} \right)^{2/3} \quad (19)$$

where  $W_{dew}$  is the actual amount of dew on the leave surfaces and  $W_{dew,max}$  is the maximum possible value ( $0.2 \text{ kgm}^{-2}$ )

#### 4.2 Stomatal resistance

The stomatal resistance  $r_s$  of a vital plant is calculated with respect to actual and maximum shortwave radiation input ( $R_{sw}$  and  $R_{sw,max}$ ) and of the available soil water content inside the root zone ( $\eta$ ) as described by Deardorff (1978):

$$r_s = r_{s,min} \left[ \frac{R_{sw,max}}{0.03R_{sw,max} + R_{sw}} + \left( \frac{\eta_{wilt}}{\eta} \right)^2 \right] \quad (20)$$

The minimum stomatal resistance  $r_{s,\min}$  depends on the type of plant and ranges from  $200 \text{ s}^{0.5}\text{m}^{-1}$  for grass up to  $400 \text{ s}^{0.5}\text{m}^{-1}$  for deciduous leafs. Alternatively to the simple Deardorff-approach, the stomata resistance can also be calculated using a photosynthesis model that allows a more dynamic description of the plant processes (Jacobs 1994).

### 4.3 Energy balance of the leaf

If the internal energy storage inside the leaf is neglected, the foliage temperature  $T_f$  can be obtained from the steady-state leaf energy budget:

$$0 = R_{\text{sw,net}}(z) + R_{\text{lw,net}}(z) - c_p \rho J_{f,h} - \rho L (J_{f,\text{evap}} + J_{f,\text{tran}}) \quad (21)$$

where  $c_p$  is the specific heat of the air and  $\rho$  the air density,  $L$  is the latent heat of vaporization.  $R_{\text{sw,net}}$  is the net shortwave radiation absorbed by the leaf surface calculated as

$$R_{\text{sw,net}}(z) = (F \cdot R_{\text{sw,dir}}(z) + R_{\text{sw,dif}}(z))(1 - a_f - tr_f)$$

Here,  $F$  is a non-dimensional parameter describing the orientation of the leafs towards the sun ( $=0.5$  for randomly orientated leafs),  $a_f$  is the albedo of the foliage and  $tr_f$  is a transmission factor (set to 0.3).

The longwave radiation budget for (21) is given by

$$R_{\text{lw,net}}(z, T_f) = \epsilon_f R_{\text{lw}}^{\downarrow}(z) + R_{\text{lw}}^{\leftrightarrow}(z) + \epsilon_f R_{\text{lw}}^{\uparrow}(z) - 2\epsilon_f \sigma_B T_f^4 - (1 - \sigma_{\text{svf}}(z))\sigma_B T_f^4$$

The source/sink terms for the atmospheric model can finally be computed using (17 a-c) with  $T_f$  obtained by solving (21):

$$Q_h(z) = \text{LAD}(z) J_{f,h} \quad (22)$$

$$Q_q(z) = \text{LAD}(z) (J_{f,\text{evapo}} + J_{f,\text{trans}}) \quad (23)$$

where LAD is the leaf area density in height  $z$ . The equations assume, that only one side of the leaf is participating in the turbulent exchange processes of heat and vapour (the luv side) and absorbs shortwave radiation, whereas in the longwave radiation spectra, both sides of the leaf take part in the radiative exchange process.

### 4.4 Water balance of the plant/soil system

To ensure a realistic simulation of the feedback mechanisms between water transpiration by the plant and water supply by the soil, the water transpired by the plant must be taken from the soil via root water uptake, resulting in a loss of soil water content. If the soil fails to supply enough water, the stomatal resistance will be increased and the transpiration rate decreases.

The total mass of water ( $m_{\text{trans}}$ ) transpired by the plant is given by the vertical integral over the transpiration fluxes in the different plant layers:

$$m_{\text{trans}} = \rho \int_0^{z_p} \text{LAD}(z) J_{f,\text{trans}}(z) dz \quad (24)$$

Following Pielkes' (1984) suggestion, the water is taken from different soil layers inside the root zone of the plant depending on the amount of roots in the layer (RAD(z) value) and the hydraulic diffusivity of the soil layer ( $D_\eta(z)$ ):

$$S_\eta(-z) = \frac{m_{trans}}{\rho_w} \left( RAD(-z) D_\eta(-z) \right) \left( \int_{-z_r}^0 RAD(-z) D_\eta(-z) dz \right)^{-1} \quad (25)$$

## 5. GROUND SURFACE AND BUILDING SURFACES

The temperature  $T_0$  of the ground surface in equilibrium can be calculated from the energy balance

$$0 = R_{sw,net} + R_{lw,net} - c_p \rho J_h^0 - \rho L \cdot J_v^0 - G \quad (26)$$

in which  $R_{sw,net}$  and  $R_{lw,net}$  are the net radiative energy fluxes,  $J_h$  and  $J_v$  are the turbulent fluxes of heat and vapour and  $G$  is the soil heat flux. In case of building surfaces (walls, roofs), the soil heat flux is replaced by the heat transmission through the wall or the roof ( $Q_w$ ) .

### 5.1 Radiative fluxes

$R_{sw,net}$  and  $R_{lw,net}$  are the net shortwave and longwave radiation absorbed by the surface calculated with respect to the temperatures of surfaces and walls „seen“ by the ground.

Using the radiative fluxes scheme introduced in section 2.4, the shortwave net flux can be written as:

$$R_{sw,net} = (R_{sw,dif}(z=0) \cos \beta + R_{sw,dif}(z=0))(1-a_s)$$

where  $\beta$  is the angle of incidence of the incoming shortwave radiation relative to the surface exposition and  $a_s$  is the surface albedo.

The calculation of the longwave net radiation must take in account the influence of potential vegetation layers above the surface as well as the longwave fluxes from buildings and reflection of radiation between buildings and the surface. For simplicity, the longwave budget is split into a fraction that is unshielded by buildings ( $R_{lw,net}^{us}$ ) and a fraction obstructed by buildings ( $R_{lw,net}^s$ ):

$$R_{lw,net}(T_0) = \sigma_{svf} R_{lw,net}^{us}(T_0) + (1 - \sigma_{svf}) R_{lw,net}^s$$

where the sky-view-factor  $\sigma_{svf}$  is used to weight the energy budget for the shielded and unshielded fraction according to the situation.

After Deardorff (1978) the exchange of longwave radiation between the ground and the vegetation (unshielded part, first term) and between the ground and buildings (shielded part, second term) can be written as:

$$\begin{aligned} R_{lw,net}^{us} &= \sigma_{lw}^\downarrow(0) \left( R_{lw}^{\downarrow,0} - \epsilon_s \sigma_B T_0^4 \right) + \left( 1 - \sigma_{lw}^\downarrow(0) \right) \frac{\epsilon_f \epsilon_s}{\epsilon_f + \epsilon_s - \epsilon_f \epsilon_s} \left( \sigma_B \bar{T}_f^4 - \sigma_B T_0^4 \right) \\ R_{lw,net}^s &= \frac{\epsilon_w \epsilon_s}{\epsilon_w + \epsilon_s - \epsilon_w \epsilon_s} \left\{ \max \left( \sigma_B \bar{T}_w^4, \sigma_B T_0^4 \right) - \sigma_B T_0^4 \right\} \end{aligned} \quad (27)$$

$\bar{T}_w$  is the average temperature of the building walls and  $\epsilon_w$  the walls' emissivity. For the shielded fraction of the energy balance it is assumed, that the energy flux from the walls is only relevant if the walls are warmer than the ground surface. If the ground surface is warmer, the reflection of the longwave radiation of the surface at the walls is the dominating effect.

In the case of building walls, the radiative scheme is less complex. Here, the effects of vegetation are neglected because only few information are available about the horizontal longwave fluxes from the vegetation layers. For vertical walls, it is assumed, that the unshielded fraction will receive 50% of the longwave radiation from the sky and the other 50% from the ground. For the shielded fraction, 2/3 of the longwave radiation are supposed to come from the emission of other walls and the remaining 1/3 of the radiation is assumed to be radiation from the ground reflected by the walls.

For roofs the radiative components are the same as for the ground surface except that  $z \neq 0$  and that additional vegetation layers above the roof are not taken into account.

## 5.2 Turbulent fluxes of sensible heat and vapour

The turbulent fluxes of heat  $J_h^0$  and vapor  $J_v^0$  at the ground surface and at building walls and roofs are calculated as

$$\begin{aligned} J_h^0 &= -K_h^0 \frac{\partial T}{\partial z} \Big|_{z=0} = -K_h^0 \frac{\theta(k=1) - T_0}{0.5\Delta z(k=1)} \\ J_v^0 &= -K_v^0 \frac{\partial q}{\partial z} \Big|_{z=0} = -K_v^0 \frac{q(k=1) - q_0}{0.5\Delta z(k=1)} \end{aligned} \quad (28 \text{ a,b})$$

where  $k=1$  indicates the first calculation layer above or adjacent to the surface and  $K_h^0, K_v^0$  are the exchange coefficients for heat and vapour between the surface and the air. Both are calculated with respect to the thermal stratification between the surface and the overlying air layer (Asaeda *et al.* 1993). In case of walls, the notations in (28 a,b) have to be adopted according to the orientation of the wall. In case low wind speeds leading to free convection conditions, the so-called  $z^{-1/3}$  law is used to describe vertical transport by thermals (Parhans and Schrodin 1980).

The Surface humidity  $q_0$  can be obtained from the soil moisture content at level  $z=-1$  using the  $\beta$ -approach from Deardorff (1978):

$$\begin{aligned} q_0 &= \beta q_*(T_0) + (1-\beta)q(z=1) \\ \beta &= \min(1, \eta(z=-1) / \eta_{fc}) \end{aligned} \quad (29)$$

where  $\eta$  is the volumetric soil water content in the first soil layer and  $\eta_{fc}$  is its value at field capacity. The water flux is linked to the soil hydraulic model using an additional sink term  $S_{\eta,0}$  related to the evaporation at the surface with

$$S_{\eta,0}(k=-1) = -\frac{\rho_w}{\rho_w} J_v^0 \frac{1}{\Delta z(k=-1)} \quad (30)$$

in which  $k=-1$  is the first layer of the soil model with the thickness  $\Delta z$  and  $\rho_w$  is the density of water. Practical application have shown that it is more realistic to distribute the water loss over the upper two layers of soil and also use these two layers to estimate  $\beta$  in (29) rather than using only the uppermost layer. Otherwise, because of the layers thinness, it will dry out too fast.

### 5.3 Soil heat flux and heat flux through building walls

The soil heat flux is calculated from the surface temperature and the temperature of the first level of the soil model below the surface:

$$G = \lambda_s(k = -1) \frac{T_0 - T(k = -1)}{0.5\Delta z(k = -1)} \quad (31)$$

where  $\lambda_s$  is the heat conductivity of the first soil layer which depends on the soil material and the water content.

For buildings, G is replaced by  $Q_w$ :

$$Q_w = k(T_w - T_{a,i}) \quad (32)$$

in which k is the heat transmission coefficient of the wall material and  $T_{a,i}$  is the air temperature inside the building. This approach is rather simple and does not take into account the heat storage inside the wall material.

## 6. NUMERICAL ASPECTS

### 6.1 Solution Techniques

The differential equations in the model are solved on a staggered grid system using the finite difference method. The three dimensional advection-diffusion equations are de-coupled using the Alternating Directions Implicit (ADI) method in combination with an upstream advection scheme. This scheme implies a relatively high numerical diffusion but allows a quick and implicit solution of the equations and has therefore been chosen in the ENVI-met model.

To solve the Navier-Stokes equations, a splitting method after Patrinos and Kistler (1977) is used. Here, the prognostic equations for a mass-conserving wind field  $u_i^{t+\Delta t}$  are split into an auxiliary flow field ( $u^{\text{aux}}$ ) and a pressure field (p):

$$\frac{\partial u_i^{t+\Delta t}}{\partial t} = \frac{\partial u_i^{\text{aux}}}{\partial t} + \frac{1}{\rho} \nabla p \quad (33)$$

The pressure variable is then removed from the prognostic equations (1 a-c) leading to a set of three prognostic equations for an auxiliary flow field:

$$\begin{aligned} \frac{\partial u^{\text{aux}}}{\partial t} + u_i \frac{\partial u^{\text{aux}}}{\partial x_i} &= K_m \left( \frac{\partial^2 u^{\text{aux}}}{\partial x_i^2} \right) + f(v - v_g) - S_u \\ \frac{\partial v^{\text{aux}}}{\partial t} + u_i \frac{\partial v^{\text{aux}}}{\partial x_i} &= K_m \left( \frac{\partial^2 v^{\text{aux}}}{\partial x_i^2} \right) - f(u - u_g) - S_v \\ \frac{\partial w^{\text{aux}}}{\partial t} + u_i \frac{\partial w^{\text{aux}}}{\partial x_i} &= K_m \left( \frac{\partial^2 w^{\text{aux}}}{\partial x_i^2} \right) + g \frac{\theta(z)}{\theta_{\text{ref}}(z)} - S_w \end{aligned} \quad (34 \text{ a-c})$$

This flow field contains the correct vorticity, but is not mass conserving, which means that it does not fulfil the filter condition (2).

The matching pressure field can be obtained by solving the Poisson equation:

$$\nabla^2 p = \frac{\rho}{\Delta t} \nabla u_i^{\text{aux}} \quad (35)$$

using the iterative Simultaneous Over Relaxation (SOR) method. Finally, the correct and approximately mass-conserving flow field can be calculated from

$$u_i^{t+\Delta t} = u_i^t - \frac{\Delta t}{\rho} \frac{\partial p}{\partial x_i} \quad (36)$$

The steep pressure gradients occurring in microscale simulations with obstacles require very small time steps to solve the set of wind field equations. Therefore, the wind field is not treated as a “normal” prognostic variable in ENVI-met, but is updated after a given time interval to take into account changes in turbulence and thermal stratification. Using the wind field as a normal variable is technically possible, but too time consuming on recent computers.

## 6.2 Computational Domain and Grid Structure

Depending on the problem, the total size of the three dimensional model X, Y and Z as well as the resolution of the grid can be selected within a wide range. By default the spacing  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  is equidistant in each direction (only the lowest grid cell above ground is normally split into 5 sub-cells with size  $\Delta z_g=0.2\Delta z$  to increase accuracy in calculating surface processes).

The three dimensional model is nested into a one-dimensional model which extends up to 2500 m height. The values of the one dimensional model are used as reference values as well as inflow profiles and top boundary conditions for the three-dimensional model.

## REFERENCES

- Asaeda et al. (1993). The subsurface transport of heat and moisture and its effect on the environment: A numerical model, *Boundary Layer Met.*, 65, 159–179
- Barden, H. (1982). Simulationsmodell für den Wasser-, Energie- und Stoffhaushalt in Pflanzenbeständen, *Rep. Inst. Met. Univ. Hanover*, 23
- Bruse, M. (1999). The influences of local environmental design on microclimate (...), Ph.D Thesis University of Bochum, Bochum (in German)
- Bruse, M. and H. Fleer (1998). Simulating Surface- Plant-Air Interactions Inside Urban Environments with a Three Dimensional Numerical Model, *Environmental Software and Modelling*, (13), S. 373–384
- Bruse, M.. (1995). Development of a microscale model for the calculation of surface temperatures in structured terrain, MSc Thesis, Inst. for Geography, Univ. Bochum,
- Clapp R. B. and G. Hornberger (1978). Empirical equations for some soil hydraulic properties, *Water Resource. Res.*, 14, 601–604
- Deardorff, J. W. (1978). Efficient prediction of ground surface temperature and moisture with inclusion of a layer of vegetation, *J. Geophys. Res.*, 83, 1889–1903
- Eichhorn, J.(1989). Entwicklung und Anwendung eines dreidimensionalen mikroskaligen Stadtklimamodells, PhD- Thesis, Univ. Mainz, D.
- Gross, G. (1991). Anwendungsmöglichkeiten mesoskaliger Simulationsmodelle dargestellt am Beispiel Darmstadt, *Meteorol. Rundschau*, 43, 97–112
- Gutman; D. P. and K. E. Torrance (1975). Response of the urban boundary layer to heat addition and surface roughness, *Boundary Layer Met.*, 9, 217–233
- Hirt, C. W. and J. L. Cook (1972). Calculating three-dimensional flows around structures and over rough terrain, *J. Comput. Phys.*, 10, 324–340

- Launder B.E. and D. B. Spalding (1974). The numerical computation of turbulent flows, *Comp. Methods Appl. Mech. Eng.*, 3, 269–289
- Liu J. et al. (1996). E- $\epsilon$  modelling of turbulent air flow downwind of a model forest edge, *Boundary Layer Met.*, 77, 21–44
- Jacobs, C. M. J. (1994). Direct impact of atmospheric CO<sub>2</sub> enrichment on regional transpiration, PhD Thesis, Wageningen Agricultural University
- Mellor G. L. and T. Yamada (1975). A simulation of the Wangara atmospheric boundary layer data, *J. Atmos. Sci.* 32, 2309–2329
- Panhans, W.-G and R. Schrodin (1980). A one-dimensional circulation and climate model and its application to the lower atmosphere, *Contrib. Atmosph. Phys.*, 53, 264–294
- Patrinos A. A. and A. L. Kistler (1976). A numerical study of the chicago lake breeze, *Boundary Layer Met.*, 12, 93–123
- Pielke R. A. (1984). *Mesoscale meteorological modelling*, Academic Press, Orlando
- Schilling, V. (1990). A parameterization for modelling the meteorological effects of tall forests - A case study of a large clearing, *Boundary Layer Met.*, 55, 283–304
- Sievers, U. et al.(1987). Numerische Simulation des urbanen Klimas mit einem zweidimensionalen Modell, *Meteorol. Rundschau* 40(1 and 3), 40–52 and 65–83,
- Stull, R. (1994). A convective transport theory for surface fluxes, *J. Atmos. Sci.*, 51, 3–22
- Taesler R. and C. Anderson (1984). A method for solar radiation computings using routine meteorological observations, *Energy and Buildings*, 7, 341–352
- Terjung W. H. and P. O' Rourke (1980). Simulating the causal elements of urban heat islands, *Boundary Layer Met.* 19, 93–188
- Tjernström, M. (1989). Some tests with a surface energy balance scheme including a bulk parameterization for vegetation in a mesoscale model, *Boundary Layer Met.*, 48, 33–68
- Watanabe, T. (1994). Bulk parameterization for a vegetated surface and its application to a simulation of nocturnal drainage flow, *Boundary Layer Met.*, 70, 13–35
- Wilson, J. D. (1988). A second order closure model for flow through vegetation, *Boundary Layer Met.* 42, 371–392
- Yamada, T. (1982). A numerical model study of turbulent airflow in and above a forest canopy, *J. Met. Soc. Japan*, 60, 439–454